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### Semester One Examination, 2020

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNIT 3**

## Section One:

## Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free 35% (52 Marks)

This section has**eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (5 marks)

A curve, defined for , passes through the point and its gradient is given by

(a) Verify that is a stationary point, determine the value of the second derivative at and hence describe the nature of the stationary point. (3 marks)

|  |
| --- |
| **Solution** |
| , so is a stationary point.  , so is a local minimum. |
| **Specific behaviours** |
| ✓ simplifies to three integers that sum to zero  ü correct value of second derivative  ü states correct nature |

(b) Determine the equation of the curve. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct antiderivative  ü evaluates constant and writes equation |

Question 2 (5 marks)

Determine the area bounded by the line and the parabola .

|  |
| --- |
| **Solution** |
| Intersect when  Bounded area |
| **Specific behaviours** |
| ✓ equates functions and simplifies  ü bounds of integral  ü writes definite integral  ü antidifferentiates  ü correct area |

Question 3 (8 marks)

Determine

(a) when . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates correct use of chain rule  ü correct derivative (any form) |

(b) when . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü or correct  ü correct derivative in terms of  ü correct value |

(c) when . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates correct use of quotient rule  ü correct derivative  ü correct value, simplified |

Question 4 (7 marks)

A bag contains counters, marked with and the remainder marked with . The random variable is the number on a randomly selected counter from the bag.

(a) Explain why is a Bernoulli random variable and determine the mean and variance of .

(3 marks)

|  |
| --- |
| **Solution** |
| is a Bernoulli random variable as it can only take on two values, and . |
| **Specific behaviours** |
| ✓ states can only take on two values  ü mean  ü variance |

Each of the students in a class randomly select a counter from the bag, note the number on the counter and then replace it back in the bag. The random variable is the number of students in the class who select a counter marked with .

(b) Define the distribution of and determine the mean and variance of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states binomial with parameters  ü mean  ü variance |

(c) Explain why it is important that the students replace their counters for the distribution of in part (b) to be valid. (1 mark)

|  |
| --- |
| **Solution** |
| If counters not replaced, the probability of a success (selecting a counter marked with ) would not remain constant. |
| **Specific behaviours** |
| ✓ indicates that probability of success must be constant |

Question 5 (7 marks)

Functions and are such that

(a) Determine . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ integrates rate of change  ü determines change  ü correct value |

(b) Use the increments formula to determine an approximation for . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ values of and  ü use of increments formula  ü correct approximation |

(c) Briefly discuss whether using the information given about and the increments formula would yield a reasonable approximation for . (1 mark)

|  |
| --- |
| **Solution** |
| No, approximation wouldn't - the change is not a small change. *(NB Yields )* |
| **Specific behaviours** |
| ✓ states no with reason |

Question 6 (5 marks)

The graph of has a stationary point at and , where is a constant.

Determine the interval over which and .

|  |
| --- |
| **Solution** |
| Concave down:  Other stationary point:  Hence when .  Required interval: . |
| **Specific behaviours** |
| ✓ value of  ü interval where  ü second stationary point  ü interval where  ü correct interval |

Question 7 (8 marks)

Initially, particle is stationary and at the origin. Particle moves in a straight line so that at time seconds its acceleration cms-2 is given by where .

(a) Determine the speed of after second. (3 marks)

|  |
| --- |
| **Solution** |
| Hence speed is cm/s. |
| **Specific behaviours** |
| ü indicates is integral of  ✓ expression for velocity  ü correct speed |

(b) Determine the speed of when it returns to the origin. (5 marks)

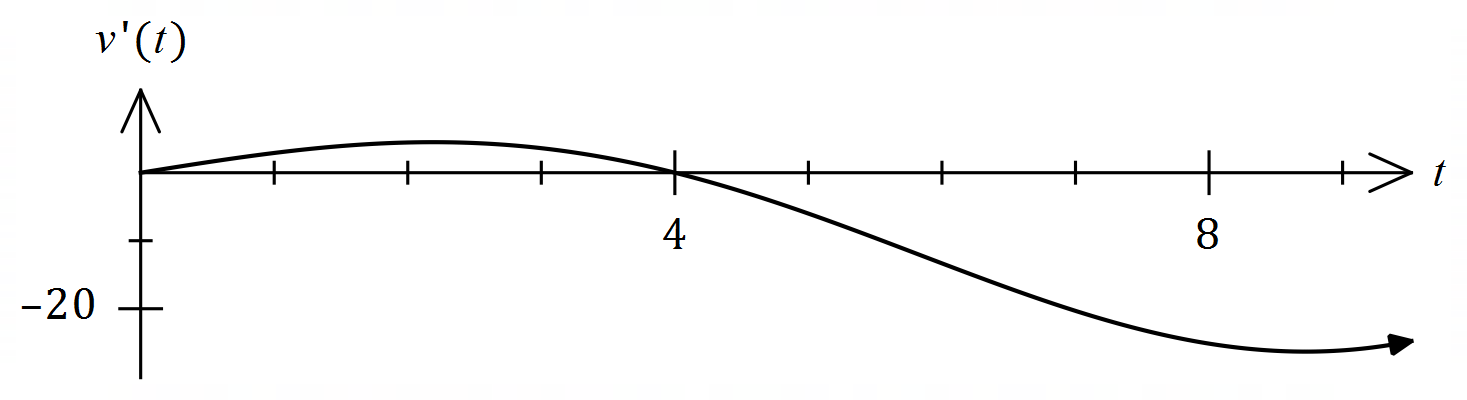
|  |
| --- |
| **Solution** |
| Require change in displacement for  Hence speed is cm/s. |
| **Specific behaviours** |
| ü obtains expression for in terms of  ü equates  ü solves for  ü obtains velocity  ü correct speed, with units |

Question 8 (7 marks)

(a) Determine an expression for . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct use of product rule  ü correct derivative |

The volume of water in a tank, litres, is changing at a rate given by , where is the time in hours. The rate of change is shown in the graph below.



(b) Using the result from part (a) or otherwise, determine the change in volume of water in the tank between and hours. (5 marks)

|  |
| --- |
| **Solution** |
| 1. Using (a):  2. And so:  3. Hence: |
| **Specific behaviours** |
| ✓ indicates required definite integral  ü line 1 - uses part (a)  ü line 2 - expression to evaluate integral  ü line 3 - antidifferentiates ready for substitution  ü correct change in volume, with units |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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